LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – STATISTICS

FIFTH SEMESTER – NOVEMBER 2007

ST 5501 - TESTING OF HYPOTHESIS

BB 15

Date : 26/10/2007 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

PART – A

(10 x 2 = 20 marks)

Answer ALL the questions.

1. Distinguish between simple and composite hypotheses. Give examples.
2. Explain the two types of errors in testing of hypothesis.
3. Define one-parameter exponential family of distributions. Show that Poisson distribution belongs to this family.
4. State monotone likelihood property.
5. Define SPRT.
6. Define LRT.
7. Write down the statistic used for testing the significance of difference of means of two normal populations for large samples.
8. State any two applications of F-distribution.
9. Distinguish between parametric tests and non-parametric tests.
10. What are the assumptions made for the applicability of non-parametric tests?

PART – B

Answer any FIVE questions. **(5 x 8 = 40 marks)**

11. Define the following terms:
    i) Test Function ii) Power function
    iii) UMP test iv) Unbiased test
12. Let $X_1, X_2, \ldots X_n$ be a random sample from b(1,p). Obtain the best critical region of size α for testing

    $$H_o : P = P_o \text{ against } H_1 : P = P_1\ (< P_o)$$
13. A random sample of size 1 is selected at random form a population with density function

    $$f(x;\theta) = \frac{1}{\theta}; \qquad 0 < x < \theta$$

    to test H_0: θ = 1 against H_1: θ = 2. H_0 is rejected if $X_1 > 0.99$.
    Find the sizes α and β of the two types of errors.
14. Show that there in no UMP test for testing H_0 : $\theta = \theta_0$ against H_1: $\theta \neq \theta_0$ where the pd f f is given by

    $$f(x;\theta) = \theta e^{-\theta x}; \qquad x > 0; \theta > 0$$
15. Examine whether the Poisson distribution with parameter θ has a monotone likelihood ratio in T = $\sum_{1}^{n} Xi$, where $X_1, X_2, \ldots X_n$ in a random sample from P(θ).

16. The probability mass function of a random variable X under Ho and H_1 are given by

x	:	1	2	3	4	5	6
$f_o(x)$	:	0.01	0.01	0.01	0.01	0.01	0.95
$f_1(x)$	:	0.05	0.04	0.03	0.02	0.01	0.85

Compute $\lambda(x)=\frac{f_1(x)}{f_0(x)}$ and obtain the most powerful test of size $\alpha=0.03$. Also obtain the power.

17. Explain the construction of confidence interval for μ with confidence coefficient 0.99 based on a random sample of size n from $N(\mu,\sigma^2)$, where σ^2 is not known.
18. Explain the median test.

PART – C

(2 x 20 = 40 marks)

Answer any TWO questions.

19. (a) State and establish Neyman – Pearson Lemma.
(b) A random sample of size 1 is taken from binomial distribution b(5, p) to test $H_0 : P = 0.5$ against $H_1: P = 0.6$ at $\alpha = 0.05$. Construct level α randomized test. Also find the size of type II error. (10+10)
20. (a) Consumption of milk by families in a township is normally distributed with $\mu = 5$ litres per day and variance $\sigma^2 = 25$ (litre)^2. However it is claimed that the consumption is more than 5 litres. A random sample of 64 families is selected in order to verity the claim. Obtain the UMP test of level $\alpha = 0.05$ and compute the power at $\mu = 6, 6.5, 7$, and 7.5.
(b) Show that the power of the test obtained by Neyman – Pearson Lemma is never less than its level in the case of testing simple H_o against simple alternative H_1.
(12+8)
21. Let $X_1, X_2, \ldots X_n$ be a random sample from $N((\mu,\sigma^2)$, μ - not known.
Derive size -α likelihood ratio test of $H_o: \sigma = \sigma_o$ against $H_1: \sigma \neq \sigma_o.$
22. (a) The sales data (in lakhs) of textile garments in six shops before and after special offers are given below:

Shop	:	1	2	3	4	5	6
Before offer	:	56	42	48	28	53	31
After offer	:	56	45	55	29	58	30

Test at 5% level whether the special offer is a success.
(b) Discuss the test of randomness of a sample. (12+8)

******